4 Statements with Predicates and Quantifiers

26. Which of the following statements are true, assuming the domain of discourse is the set of real numbers?

(i) $(\forall x)(\exists y)(x + y = 0)$.

(ii) $(\exists x)(\forall y)(x + y = 0)$.

(iii) $(\exists x)(\exists y)(x^2 + y^2 = -1)$.

(iv) $(\forall x)[x > 0 \Rightarrow (\exists y)(y < 0 \wedge xy > 0)]$.

27. (i) Express the compound statement $A \Leftrightarrow B$ using only the basic connectives $\neg$ and $\wedge$.

(ii) Recall that $(\exists! x)(x \in \mathbb{R} \wedge \varphi(x))$ is read as "There exists exactly one x in $\mathbb{R}$ such that $\varphi(x)$ holds". Rewrite the following statement

$$(\exists! a)(a \in \mathbb{R} \wedge a > 3 \wedge a \leq 4)$$

without using the shorthand '$\exists!$'—that is, instead of using the quantifier $\exists!$, use the quantifiers $\exists$ and $\forall$. Determine the truth value of the given statement.

28. (a) Given the table on the right, find D.

(b) The law of trichotomy states: "For every pair of real numbers a, b, exactly one of the following is true: $a = b$, $a < b$, or $a > b$". Write the law of trichotomy using quantifiers and connectives $(\forall, \exists, \wedge, \vee, \neg, \Rightarrow)$.

p	q	r	D
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	1
0	0	0	0

29. The set A is defined as follows:

$$A = \{n \in \mathbb{N} \mid n \text{ is an odd number } \wedge (\exists k)\big(k \in \mathbb{N} \wedge n = k(k+1)\big)\}.$$

Show that A is an empty set.

30. Negate the following statement with quantifiers:

$$(\forall a)(\forall b)((a^2 + b^2 = 0) \Rightarrow (a = 0) \vee (b = 0)).$$

Determine whether the negation is a true statement.

31. Express the following statement with quantifiers and prove it: There does not exist an odd number that can be expressed in the form $4j + 1$ and $4k - 1$ for integers j and k.

All above math problems are taken from the following website:
https://osebje.famnit.upr.si/~penjic/teaching.html.
THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.